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Modeling and Optimization of Performance of Four Stroke Spark Ignition Injector Engine

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Abstract— The performance of an engine whose basic design parameters are known can be predicted with the assistance of simulation programs into the less time, cost and near value of actual. This paper presents a comprehensive mathematical model of the performance parameters of four stroke spark ignition engine. The essence of this research work is to develop a mathematical model for the analysis of engine performance parameter of four stroke spark ignition engine before embarking on full scale construction, this will ensure that only optimal parameters are in the design and development of an engine and also allow to check and develop the design of engine and it's operation alternatives in an inexpensive way and less time, instead of using experimental method which requires costly research test beds. To achieve this, equations were derived which describe the performance parameters (sfc, thermal efficiency, mep, and A/F). the equations were used to simulate and optimize the engine performance of the model for various engine speeds. The optimal valves obtained for the developed bivariate mathematical models are: sfc is 0.2833kg/kwh, efficiency is 28.77% and a/f is 20.75.

Index Terms— Bivariate models, Engine performance, Injector engine, Optimization, Performance parameters, Simulation, Spark ignition

1 INTRODUCTION

TOday simulation and optimization are important domains which attract many researchers from several fields and disciplines [1]. In recent years, extensive research has been conducted in the area of simulation to model large complex systems and understand their structures and behaviors. At the same time, a variety of design principles and approaches for computer based simulation has evolved.

This paper will deal with the modeling and optimization of performance of four stroke spark ignition engine. Performance evaluation of automotive engines is of great importance for their economic operation. The method or criteria for assessing the engine performance include the determination of engine power, engine efficiency, and fuel consumption, considering the engine stroke, engine speed, mean effective pressure and bore. All of these affect the horse power, engine efficiency and its performance.

High engine efficiency means obtaining the greatest possible power with lowest possible fuel cost or lowest fuel consumption [2]. At the design and development stage an engineer would design an engine with certain aims in his mind. The aims may include the variables like indicated power, brake power, brake specific fuel consumption, exhaust emissions, cooling of engine, maintenance free operation etc. The other task of the development engineer is to reduce the cost and improve power output and reliability of an engine. In trying to achieve these goals he has to try various design concepts. After the design the parts of the engine are manufactured for the dimensions and surface finish and may be with certain tolerances. In order to verify the designed and developed engine one has to go for testing and performance evaluation of the engines.

Thus, in general, a development engineer will have to conduct a wide variety of engine tests starting from simple fuel and air-flow measurements to taking of complicated injector needle lift diagrams, swirl patterns and photographs of the burning process in the combustion chamber [3]. The nature and the type of the tests to be conducted depend upon various factors, some of which are: the degree of development of the particular design, the accuracy required, the funds available, the nature of the manufacturing company, and its design strategy.

Engine tests experiment require costly research engine test beds and skilful technicians to run them. In order to reduce the cost and time of engine design and development, it, therefore, becomes necessary to develop a computer simulation program using mathematical model that will describe and quantify engine performance process. This process is enhanced by using a computer to simulate and optimize engine performance before embarking on full scale construction. It will ensure that only optimal parameters are in the design and development of an engine. This would reduce the cost and time of engine design and development to a minimum.

This research work on the modeling, simulation and optimization of performance of four stroke spark ignition injector engine intends to develop a computer simulation program or models for simulating and optimizing engine performance before embarking on full scale construction, and to develop bivariate mathematical models and optimize the engine performance of the model various engine speeds. This research work is narrowed down to four stroke spark ignition injector engine. MATLAB toolbox library equation was used in the development of the simulation models. General non-linear multivariate least squares modeling of experimental result was carried resulting in specific bivariate models for various engine performance measures. This enabled analytical

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optimization of the engine performance using the tool of partial differentiation. Optimization is carried out on each of air-fuel ratio, SFC and thermal efficiency because the surface plots of their bivariate models revealed existence of curvature.

2 Mathematical Model of the System

2.1 Theoretical Terms

- **Torque, T (Nm):** The total effective load on the torque arm is the sum of the added weights plus the spring balance reading. Generally, the value of torque, T is the total weight applied multiplied by the torque arm length, L.

$$\text{Torque, T} = \frac{W+S}{1000}L \quad (1)$$

  Where; T = Torque (Nm), (W+S) = total weight applied, Newton, L = torque arm Length or radius arm (m).
- **Power (kw):** The power or rate of doing work is measured in watts (Nm/sec) or kilowatts and is defined as the torque multiplied by the angular velocity.

$$\text{Power (p)} = T\omega \quad (2)$$

  Where; T = torque, ω = Average speed = $(2\pi N)/60$, N = speed in rpm.
- **Fuel Consumption (kg/h):** Having timed the consumption of a control volume of fuel, the fuel consumption in kg/h may be calculated as follows. For a control volume of 100cc the consumption per second is given by $100/t$ where t is the measured time.

$$\text{Fuel consumption} = V/t \quad (3)$$

- **Specific Fuel Consumption (kg/kwh):** An important characteristic of an internal combustion engine is the specific fuel consumption which relates the thermal efficiency of the engine. This is defined as the weight of fuel required to generate each kilowatts hour of energy.

$$\text{SfC} = \text{weight of fuel/power} \quad (4)$$

- **Mass flow rate of fuel, M_F:** This is the quantity of fuel consumed in kg/s or weight of fuel supplied in kg per second.

$$M_F = \frac{(\text{volume in litre x specific gravity})}{(\text{time in sec x 1000})} \quad (5)$$

- **Efficiency, η:** This is the ratio of brake power to energy supplied by the fuel.

$$\eta = BP/(Mf \times \text{Calorific value}) \quad (6)$$

- **Brake Mean Effective Pressure:** Mean effective pressure is defined as hypothetical pressure which is thought to be acting on the piston throughout the power stroke. If it is based on brake power it is called brake mean effective pressure.

$$\text{B.M.E.P.} = \frac{bp}{LAn} = \frac{60 \times 1000 \times bp}{LAnK} = \frac{60000 \times BP}{LAnK} \quad (7)$$

2.2 Model Description

The laws governing real systems cannot result from pure analytical modeling because of complications of non-linearity. For this reason approximate system-specific models are derived to obey experimentally determined system responses. The approximate models are always determined on the criterion that error between experimental and mathematical responses must be minimum. The least squares approximation theory is chosen to be a criterion for mathematical modeling in this work.

2.3 Bivariate Quadratic Least Squares Model of the System Responses

Generally speaking least squares approximation of a response from a set of scattered data is based on minimizing square of experimental error. The generalized presentation of non-linear least squares approximation theory that is found very applicable in bivariate analysis presented here is seen in [4]. Suppose provided are n experimental responses z_i at positions $\boldsymbol{x}_i$ in a real d-dimensional parameter space, that is the vector $\boldsymbol{x}_i \epsilon R^d$ where $i\epsilon[1, 2, \ldots..n]$. The least squares method seeks to fix a function $z(\boldsymbol{x})$ that approximates the experimental responses z_i by minimizing the sum of squares of Euclidian error norms$\|z(\boldsymbol{x}_i) - z_i\|$. The error functional thus reads:

$$E_{NLS} = \sum_{i=1}^{n} \|z(\boldsymbol{x}_i) - z_i\|^2 \quad (8)$$

The approximation polynomial is generally given in the form

$$z(\boldsymbol{x}) = [\boldsymbol{a}(\boldsymbol{x})]^T \boldsymbol{b} \quad (9)$$

Where; $\boldsymbol{a}(\boldsymbol{x}) = \{a_1(\boldsymbol{x})\ a_2(\boldsymbol{x}) \ldots\ldots a_r(\boldsymbol{x})\}^T$ is the polynomial of basis vector and $\boldsymbol{b} = \{b_1\ b_2 \ldots\ldots b_r\}^T$ is the vector of coefficients of approximation function $z(\boldsymbol{x})$. For illustration the quadratic bivariate approximation function $z(\boldsymbol{x})$ has dimension $d = 2$ and order $p = 2$ such that

$$z(\boldsymbol{x}) = z(x, y) = b_1 + b_2x + b_3y + b_4x^2 + b_5xy + b_6y^2 \quad (10a)$$

$$z(\boldsymbol{x}) = \{1\ x\ y\ x^2\ xy\ y^2\} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix} \quad (10b)$$

From which it is seen that $\boldsymbol{a}(\boldsymbol{x}) = \{1\ x\ y\ x^2\ xy\ y^2\}^T$ and $\boldsymbol{b} = \{b_1\ b_2\ b_3\ b_4\ b_5\ b_6\}^T$ and $r = 6$. As shown on the works [5-6], the length or number of elements in $\boldsymbol{a}(\boldsymbol{x})$ or $\boldsymbol{b}$ is generally given as $r = \frac{(d+p)!}{d!p!}$. At the experimental points Eq.(2) which reads $z(\boldsymbol{x}) = [\boldsymbol{a}(\boldsymbol{x}_i)]^T \boldsymbol{b}$ is inserted in Eq.(1) to give:

$$E_{NLS}(\boldsymbol{b}) = \sum_{i=1}^{n} \|[\boldsymbol{a}(\boldsymbol{x}_i)]^T \boldsymbol{b} - z_i\|^2 \quad (11)$$

The coefficient vector $\boldsymbol{b}$ that minimizes the error functional $E_{NLS}(\boldsymbol{b})$ is determined by differentiating $E_{NLS}(\boldsymbol{b})$ [7] with respect to $\boldsymbol{b}$ and equated to zero to give:

$$\sum_{i=1}^{n} 2\boldsymbol{a}(\boldsymbol{x}_i)\{[\boldsymbol{a}(\boldsymbol{x}_i)]^T \boldsymbol{b} - z_i\} = \boldsymbol{0} \quad (12a)$$

This gives: $\sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i)[\boldsymbol{a}(\boldsymbol{x}_i)]^T \boldsymbol{b} - \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i) z_i = \boldsymbol{0}$ **(12b)**

Eq. (5) is re-arranged to give the minimum-error coefficient vector $\boldsymbol{b}$ as: $\boldsymbol{b} = \{\sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i)[\boldsymbol{a}(\boldsymbol{x}_i)]^T\}^{-1} \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i) z_i$ **(13)**

The minimum-error coefficient vector $\boldsymbol{b}$ as given in Eq.(13) is inserted in Eq.(9) to give the needed approximation polynomial as:

$$z(\boldsymbol{x}) = [\boldsymbol{a}(\boldsymbol{x})]^T \{\sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i)[\boldsymbol{a}(\boldsymbol{x}_i)]^T\}^{-1} \sum_{i=1}^{n} \boldsymbol{a}(\boldsymbol{x}_i) z_i \quad (14)$$

For the quadratic bivariate approximation the function $z(\boldsymbol{x})$ becomes:

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$$z(\boldsymbol{x}) = \{1\ x\ y\ x^2\ xy\ y^2\}\left[\sum_{i=1}^{n}\begin{Bmatrix}1\\ x_i\\ y_i\\ x_i^2\\ x_iy_i\\ y_i^2\end{Bmatrix}\{1\ x_i\ y_i\ x_i^2\ x_iy_i\ y_i^2\}\right]^{-1}\sum_{i=1}^{n}\begin{Bmatrix}1\\ x_i\\ y_i\\ x_i^2\\ x_iy_i\\ y_i^2\end{Bmatrix}z_i \qquad \textbf{(14a)}$$

This on multiplying out gives

$$z(\boldsymbol{x}) = \{1\ x\ y\ x^2\ xy\ y^2\}\left[\sum_{i=1}^{n}\begin{bmatrix}1 & x_i & y_i & x_i^2 & x_iy_i & y_i^2\\ x_i & x_i^2 & x_iy_i & x_i^3 & x_i^2y_i & x_iy_i^2\\ y_i & x_iy_i & y_i^2 & x_i^2y_i & x_iy_i^2 & y_i^3\\ x_i^2 & x_i^3 & x_i^2y_i & x_i^4 & x_i^3y_i & x_i^2y_i^2\\ x_iy_i & x_i^2y_i & x_iy_i^2 & x_i^2y_i & x_i^2y_i^2 & x_iy_i^3\\ y_i^2 & x_iy_i^2 & y_i^3 & x_i^2y_i^2 & x_iy_i^3 & y_i^4\end{bmatrix}\right]^{-1}\sum_{i=1}^{n}\begin{Bmatrix}1\\ x_i\\ y_i\\ x_i^2\\ x_iy_i\\ y_i^2\end{Bmatrix}z_i \qquad \textbf{(15)}$$

Eq.**(15)** is used in the modeling and optimization of engine performances as presented below. In the engine performance test experiment, the rotational speed [rev/min] designated x and torque [Nm] designated y are the independent variables while each of brake power [Kw], time for consumption of 100cc [s], air-fuel ratio, specific fuel consumption (SFC) [Kg/kwh], thermal efficiency and mean effective pressure (MEP) [bar] are the responses designated $z(x,y)$. In order to make use of Eq.(15) for each of the responses $z(x,y)$ a summation table is formed as shown in table.1.

2.4 Methodoly for use of the System Model

The idea of general non-linear multivariate least squares regression is used in generating a novel bivariate approximation of the studied system as presented in Eq.(15)
The general quadratic bivariate model has the form:

$$z(x,y) = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5xy^2 \qquad \textbf{(16a)}$$

This can be put in the vector form:

$$z(\boldsymbol{x},\boldsymbol{y}) = \{1, x, y, x^2, xy, y^2\}\begin{Bmatrix}a_0\\ a_1\\ a_2\\ a_3\\ a_4\\ a_5\end{Bmatrix} \qquad \textbf{(16b)}$$

It must be noted that the right hand side of Eq.(15) with subscripts i is numerically calculated from the experimental values to give the numerical coefficients $a_0, a_1, a_2, a_3, a_4, a_5, a_6$.
The values of table 1 are summation values calculated from experimental data and are used in the subscripted part of Eq.(15) to generate the model coefficients ($a_0, a_1, a_2, a_3, a_4, a_5, a_6$).
The accuracy of table 1 is absolutely guaranteed. For example the summation associated with $x(speed)$ in table 1 is 21000 which is derived from experimental results by summing elements of first column as follows $4500 + 4000 + 3500 + 3000 + 2500 + 2000 + 1500 = 21000$

All the bivariate models are calculated from Eq.(15) by replacing the subscripted quantities with experimental summation values in table.1 leading to the generation of relevant coefficients ($a_0, a_1, a_2, a_3, a_4, a_5, a_6$) of the models. The meaning of the surface plots (Figs.1a–1f) are obvious from looking at the axis of the plots. Considering the brake power for example, the bivariate model resulting from Eq.(15) is Eq.(16) in which $\boldsymbol{x}$ designates speed, y designates torque and z designates the brake power. The brake power (on vertical axis) is then plotted against both $x(speed)$ and y (torque) [on the horizontal axis].

This is simply done by plotting Eq.(16) in which $\boldsymbol{x}$ and y on the horizontal plane constitute the independent variables and z is the modeled dependent variable. The same thing is done for all the other bivariate models.

TABLE 1
SUMMATION TABLE FOR THE EXPERIMENTAL COODINATES

Elements of the coefficient matrix	Sum of elements of the coefficient matrix
1	7
x	21000
y	564.50
x^2	70000000
xy	1662000
y^2	$4.571975e+004$
x^3	$2.52e+011$
x^2y	$5.43975e+009$
xy^2	132201000
y^3	$3.717951875e+006$
x^4	$9.5725e+014$
x^3y	$1.92735e+013$
x^2y^2	$4.24932625e+011$
xy^3	$1.05669195e+010$
y^4	$3.034938021875e+008$

Making use of Eq.(15) and table.1 the following bivariate models and surfaces for the engine responses are calculated.

3 RESULTS AND OPTIMIZATION

3.1 Bivariate Model for Brake Power

$$z(x,y) = -8.1458 + 1.4851\times10^{-3}x + 0.17215y - 9.1691\times10^{-8}x^2 + 9.1845\times10^{-5}xy - 9.5256\times10^{-4}y^2 \qquad \textbf{(17)}$$

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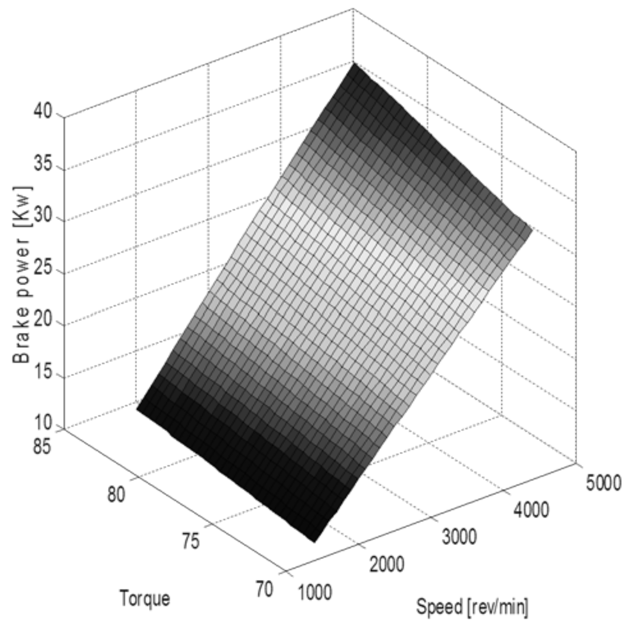

Fig.1a: The Brake power approximation surface

3.2 Bivariate Model for Brake Time

$$z(x,y) = 1073.7 - 0.15266x - 19.101y + 5.6658 \times 10^{-6}x^2 + 1.3286 \times 10^{-3}xy + 9.0557 \times 10^{-2}y^2 \quad \textbf{(18)}$$

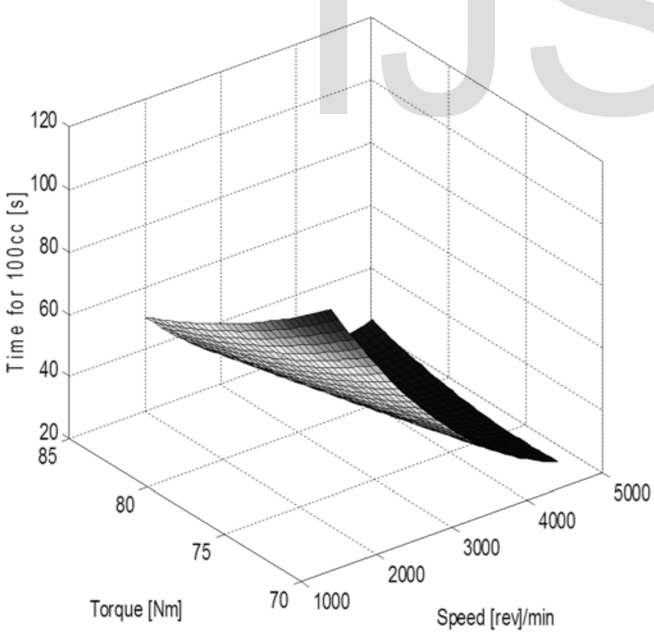

Fig.1b: The approximation surface for Time of consumption of 100cc

3.3 Bivariate Model for Air/Fuel Ratio

$$z(x,y) = 42.487 + 0.021151x - 0.47443y - 3.0075 \times 10^{-6}x^2 - 9.024 \times 10^{-5}xy - 1.7503 \times 10^{-4}y^2 \quad \textbf{(19)}$$

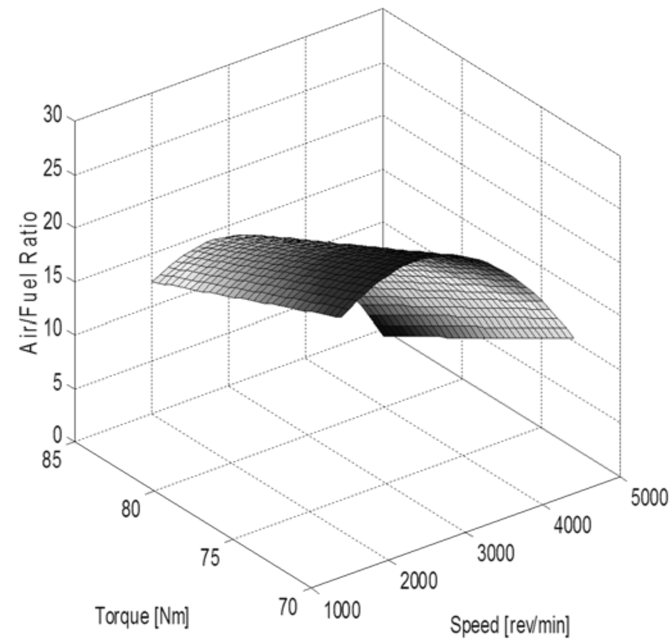

Fig.1c: The approximation surface for a/f ratio

3.4 Bivariate Model for Brake SFC

$$z(x,y) = -3.2038 + 3.1904 \times 10^{-4}x + 0.075294y + 4.4981 \times 10^{-9}x^2 - 4.4705 \times 10^{-6}xy - 3.8412 \times 10^{-4}y^2 \quad \textbf{(20)}$$

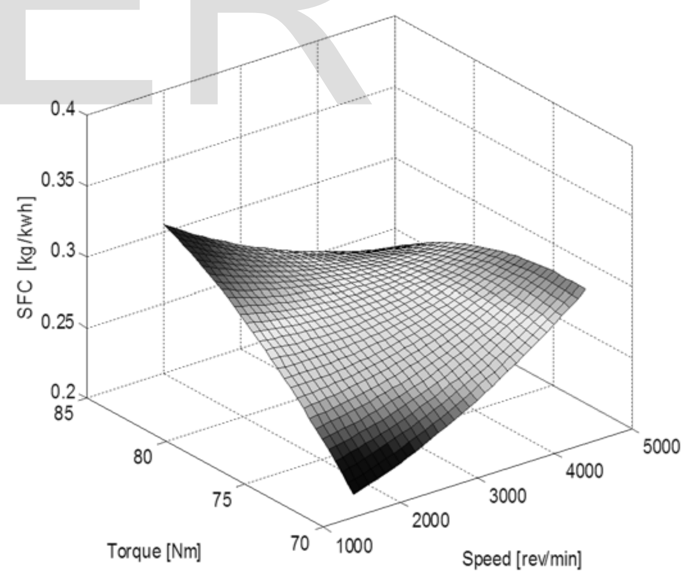

Fig.1d: The approximation surface for SFC

3.5 Bivariate Model for Thermal Efficiency

$$z(x,y) = 358.06 - 0.030482x - 7.0822y - 3.9673 \times 10^{-7}x^2 + 4.2305 \times 10^{-4}xy + 0.036024y^2 \quad \textbf{(21)}$$

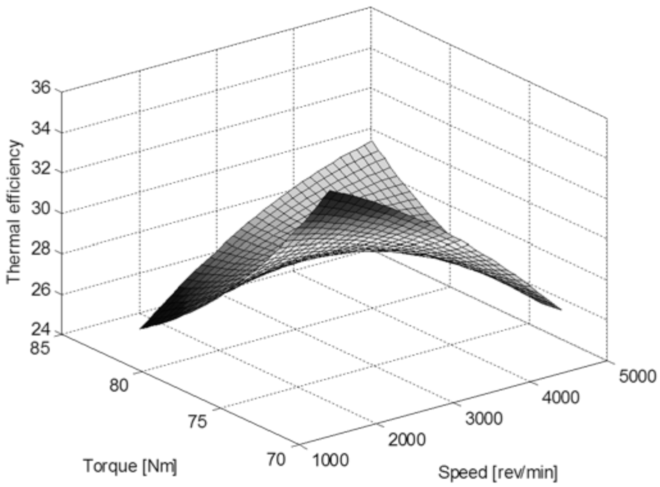

Fig.1e: The approximation surface for Thermal Efficiency

3.6 Bivariate Model for MEP

$$z(x,y) = -15.399 + 2.4873\times10^{-3}x + 0.71472y - 1.4104\times10^{-7}x^2 - 2.2260\times10^{-5}xy - 1.8034\times10^{-3}y^2 \quad \textbf{(22)}$$

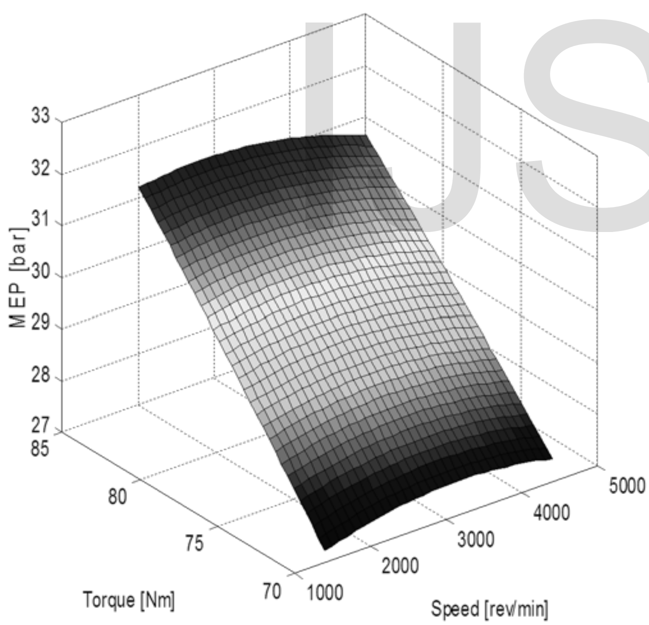

Fig.1f: The approximation surface for MEP

3.7 Optimization

An optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function [8-9]. It has been stated earlier in the existing text that the brake thermal efficiency for spark ignition engine which this work examines is between 20 and 30 percent [10]. Eligibility of each of the system's responses for optimization is based on the configuration of the approximate surface. It is seen from the surfaces for Brake power and mean effective pressure that there is no question for optimization being that linear approximation could have given an adequately accurate result. Judging from the behavior of the generated approximation surfaces the candidate for optimization are air-fuel ratio, SFC and thermal efficiency. Of particular interest is the thermal efficiency for which the optimization is done first. The generated bivariate model is:

$$z(x,y) = 358.06 - 0.030482x - 7.0822y - 3.9673\times10^{-7}x^2 + 4.2305\times10^{-4}xy + 0.036024y^2 \quad \textbf{(23)}$$

Where it should be recalled that x represents speed, y represents torque and $z(x,y)$ represents thermal efficiency. It is seen from **Fig.1e.** that a clear cut optimal exist at which the theory of partial differentiation gives $\frac{\partial}{\partial x}z(x,y) = 0$ and $\frac{\partial}{\partial y}z(x,y) = 0$. As seen in Fig.1e this point has lifted wings due to experimental error because (x,y) should vary simultaneously. This point should have depressed wings for it to be a point of maximum efficiency. Optimality thus satisfies the pair of equations

$$\frac{\partial}{\partial x}z(x,y) = -0.030482 - 2\times3.9673\times10^{-7}x + 4.2305\times10^{-4}y = 0 \quad \textbf{(24a)}$$

$$\frac{\partial}{\partial y}z(x,y) = -7.0822 + 4.2305\times10^{-4}x + 2\times0.036024y = 0 \quad \textbf{(24b)}$$

Eq.(24) is rearranged into a system of linear equations:

$$\boldsymbol{Px} = \boldsymbol{p} \quad \textbf{(25)}$$

Where $\boldsymbol{P}$ is a 2 by 2 symmetric matrix given as $\begin{bmatrix} -2\times3.9673\times10^{-7} & 4.2305\times10^{-4} \\ 4.2305\times10^{-4} & 2\times0.036024 \end{bmatrix}$ and $\boldsymbol{p} = \begin{Bmatrix} 0.030482 \\ 7.0822 \end{Bmatrix}$.

Matrix inversion gives the optimal point as follows:

$$\boldsymbol{x} = \boldsymbol{P}^{-1}\boldsymbol{p} = \begin{bmatrix} -2\times3.9673\times10^{-7} & 4.2305\times10^{-4} \\ 4.2305\times10^{-4} & 2\times0.036024 \end{bmatrix}^{-1} \begin{Bmatrix} 0.030482 \\ 7.0822 \end{Bmatrix},$$

$$\boldsymbol{x} = \begin{Bmatrix} 3387.7\text{rev/min} \\ 78.407\text{Nm} \end{Bmatrix}$$

This is the point of maximum thermal efficiency. When this optimal point is inserted into Eq.(21), the maximum thermal efficiency becomes 28.777 percent.

The model generated for SFC is:

$$z(x,y) = -3.2038 + 3.1904\times10^{-4}x + 0.075294y + 4.4981\times10^{-9}x^2 - 4.4705\times10^{-6}xy - 3.8412\times10^{-4}y^2 \quad \textbf{(26)}$$

From **Fig.1d** it can be seen that optimality exists when

$$\frac{\partial}{\partial x}z(x,y) = 3.1904\times10^{-4} + 2\times4.4981\times10^{-9}x - 4.4705\times10^{-6}y = 0 \quad \textbf{(27)}$$

$$\frac{\partial}{\partial y}z(x,y) = 0.075294 - 4.4705\times10^{-6}x - 2\times3.8412\times10^{-4}y = 0 \quad \textbf{(28)}$$

It is seen from Eqs.(27) and (28) that $\boldsymbol{P} = \begin{bmatrix} 2\times4.4981\times10^{-9} & -4.4705\times10^{-6} \\ -4.4705\times10^{-6} & -2\times3.8412\times10^{-4} \end{bmatrix}$, $\boldsymbol{P} = \begin{Bmatrix} -3.1904\times10^{-4} \\ -0.075294 \end{Bmatrix}$. Matrix inversion gives the optimal point as follows:

$$\boldsymbol{x} = \boldsymbol{P}^{-1}\boldsymbol{p} = \begin{bmatrix} 2\times4.4981\times10^{-9} & -4.4705\times10^{-6} \\ -4.4705\times10^{-6} & -2\times3.8412\times10^{-4} \end{bmatrix}^{-1} \begin{Bmatrix} -3.1904\times10^{-4} \\ -0.075294 \end{Bmatrix},$$

$$\boldsymbol{x} = \begin{Bmatrix} 3402\text{ rev/min} \\ 78.212\text{ Nm} \end{Bmatrix}$$

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When this point is inserted into Eq.(20) optimal SFC=0.28336 results. This is the optimal value for SFC.

The model for air-fuel ratio is seen from Eq.(19) to be:

$$z(x,y) = 42.487 + 0.021151x - 0.47443y - 3.0075 \times 10^{-6}x^2 - 9.024 \times 10^{-5}xy - 1.7503 \times 10^{-4}y^2 \quad \textbf{(29)}$$

It is clearly seen from the air –fuel ratio surface of **Fig.1c** that optimality that is approximately constant with torque exists at some value of speed. Thus the idea of optimization used here is to keep Torque fixed and differentiate with respect to speed to give

$$\frac{\partial}{\partial x}z(x,y) = 0.021151 - 2 \times 3.0075 \times 10^{-6}x - 9.024 \times 10^{-5}y = 0 \quad \textbf{(30)}$$

This is rearranged to give: $x = \frac{0.021151 - 9.024 \times 10^{-5}y}{2 \times 3.0075 \times 10^{-6}}$ **(31)**

A convenient value for y is then chosen to give an optimal location for air-fuel ratio. Since each of thermal efficiency and SFC has clear cut optimal locations that are very close to each other it is recommended that y value to be inserted in Eq.(31) can be located at either the arithmetic or geometric mean of the optimal locations of thermal efficiency and SFC. The arithmetic or geometric means of the optimal locations of thermal efficiency and SFC are respectively given by

$$\boldsymbol{x}_{am} = \frac{1}{2}\left(\begin{Bmatrix} 3387.7\text{rev/min} \\ 78.407\text{Nm} \end{Bmatrix} + \begin{Bmatrix} 3402\text{ rev/min} \\ 78.212\text{ Nm} \end{Bmatrix}\right) = \begin{Bmatrix} 3394.8\text{rev/min} \\ 78.31\text{Nm} \end{Bmatrix}$$

$$\boldsymbol{x}_{gm} = \sqrt{\begin{Bmatrix} 3387.7\text{rev/min} \\ 78.407\text{Nm} \end{Bmatrix}\begin{Bmatrix} 3402\text{ rev/min} \\ 78.212\text{ Nm} \end{Bmatrix}} = \begin{Bmatrix} 3394.8\text{ rev/min} \\ 78.309\text{ Nm} \end{Bmatrix}$$

It is seen that both arithmetic and geometric means of the optimal locations of thermal efficiency and SFC are almost identical. Then $y = 78.31$ say is inserted into Eq.(31) to give $x = 2341.5$. The optimal air-fuel ratio of 20.75 occurs at $\boldsymbol{x} = \begin{Bmatrix} 2341.5\text{ rev/min} \\ 78.31\text{ Nm} \end{Bmatrix}$.

4 Conclusion

A bivariate mathematical model for the analysis of engine Performance parameters of four stroke spark ignition engine was developed. The general non-linear multivariate least square modeling was carried out resulting in specific bivariate models for various engine performance measures. This enabled analytical optimization of the engine performance using the tool of partial differentiation. Optimization is carried out on each of air-fuel ratio, SFC and thermal efficiency because the surface plots of their bivariate models revealed existence of curvature. Optimization yielded that maximum thermal efficiency exists at the point $\begin{Bmatrix} Speed \\ Torque \end{Bmatrix} = \begin{Bmatrix} 3387.7\text{rev/min} \\ 78.407\text{Nm} \end{Bmatrix}$ as 28.777 percent. Accuracy and reliability of this optimum must be noted being that it has been stated earlier in the existing text that the brake thermal efficiency for spark ignition engine which this work examines is between 20 and 30 percent [11]. The optimal value for the SFC is calculated as SFC=0.28336 at the poin $\begin{Bmatrix} Speed \\ Torque \end{Bmatrix} = \begin{Bmatrix} 3402\text{ rev/min} \\ 78.212\text{ Nm} \end{Bmatrix}$. Optimal air-fuel ratio of 20.75 computed for the engine to occur at $\begin{Bmatrix} Speed \\ Torque \end{Bmatrix} = \begin{Bmatrix} 2341.5\text{ rev/min} \\ 78.31\text{ Nm} \end{Bmatrix}$. It must be pointed out that the results of the generated bivariate models for various engine performance correlate closely with those of both experiment and MATLAB simulation. Based on the achievements or contributions listed above, it can be concluded that this research work has attained its set objectives.

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